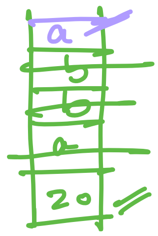
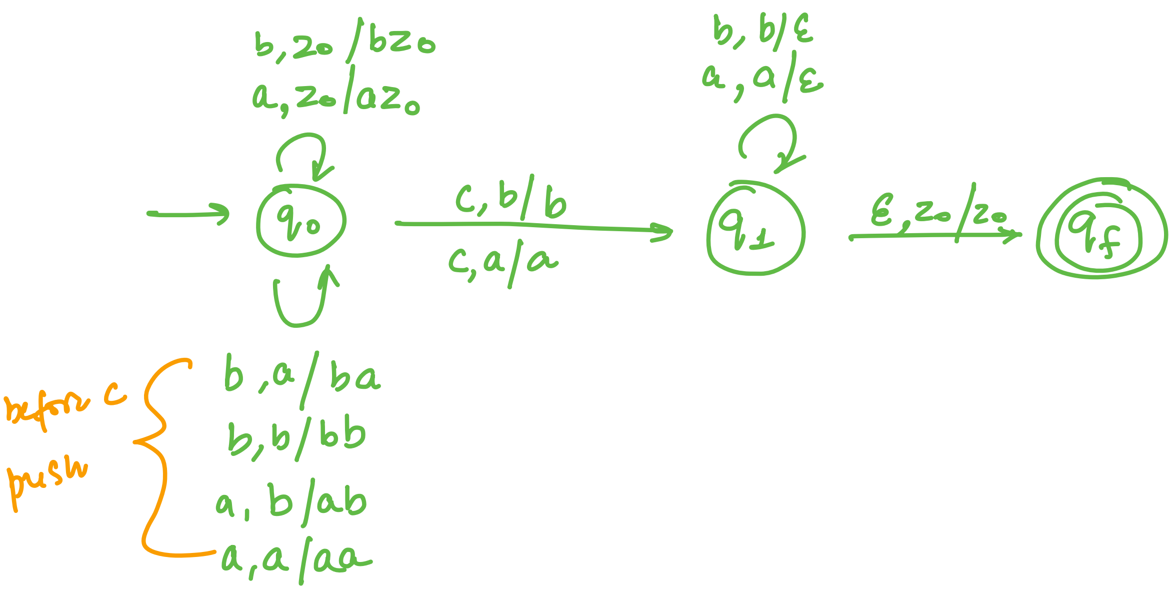
Eg: $wcw^R \mid w \in (a,b)^+$

$\underbrace{abb}_{x}\ a\ c\ \underbrace{abba}_{x}\ \varepsilon$ } Palindromic strings of odd length

$2x+1$

push

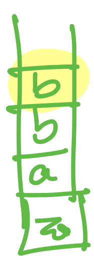


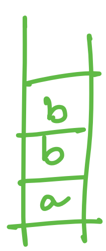


Eg: $ww^R \mid w \in (a,b)^+$ } Even length palindromic strings

$\underbrace{abb}_{w}\ \underbrace{bba}_{w^R}$

NPDA

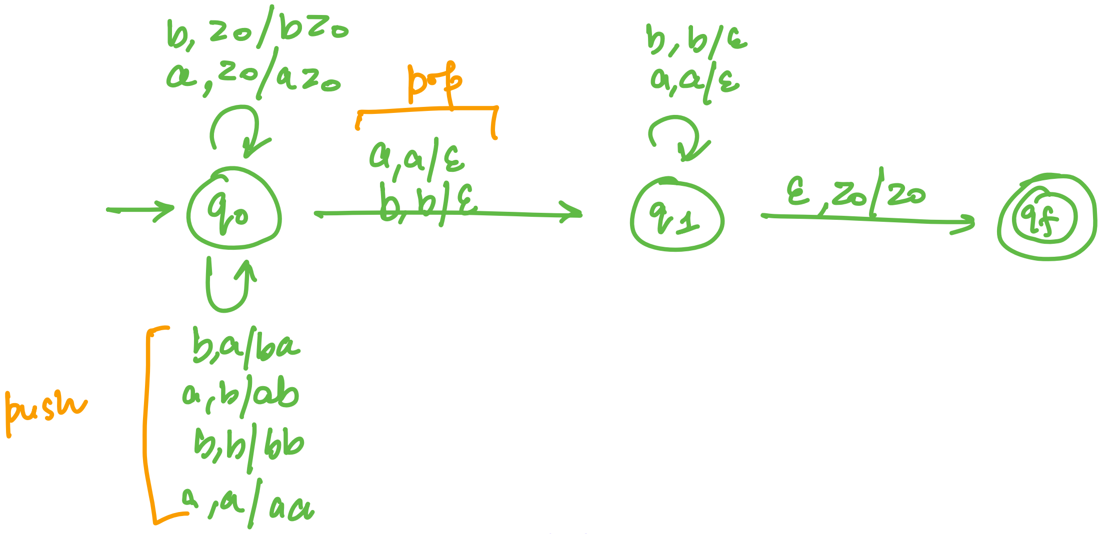


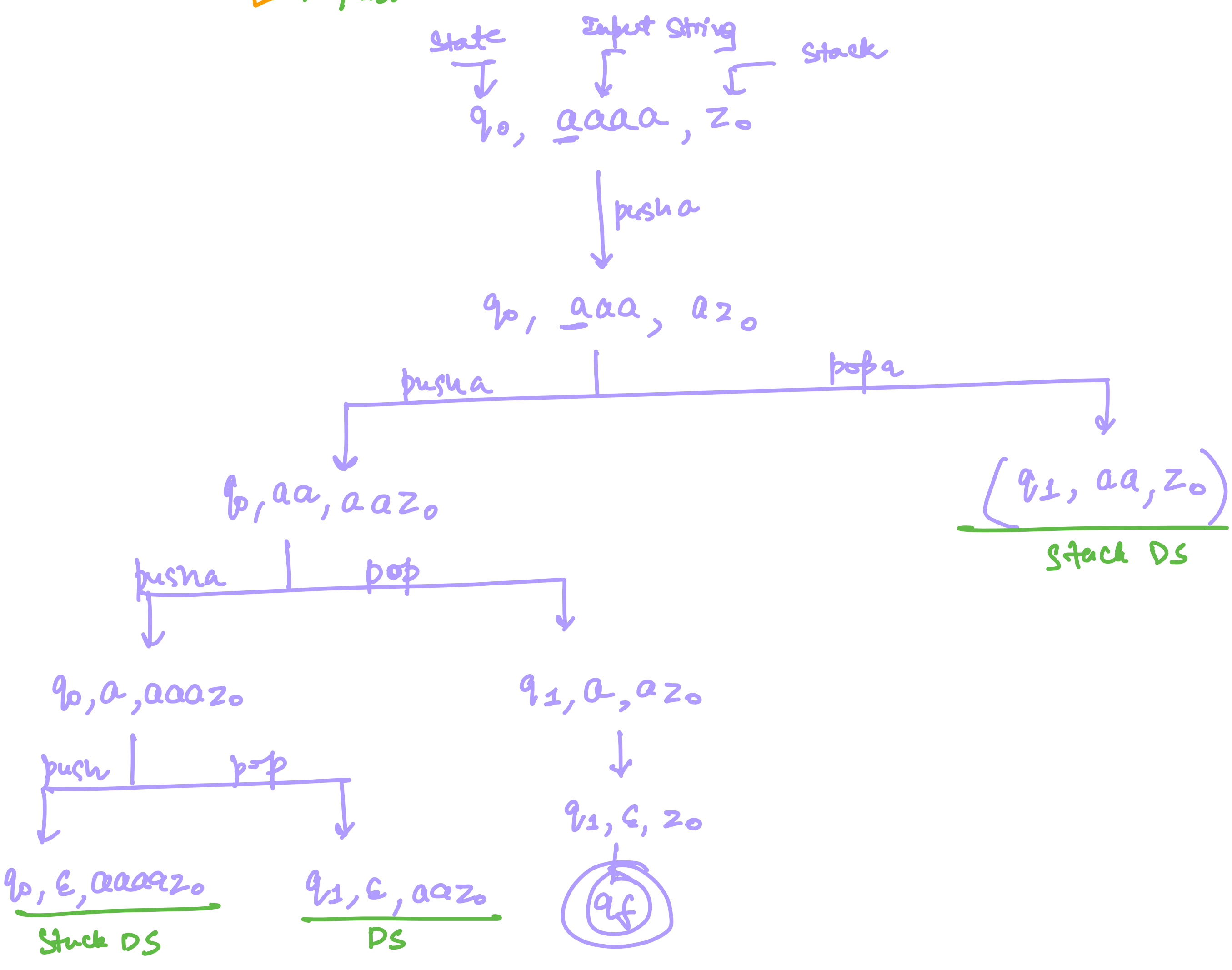


input alphabet, stack top alphabet

push

pop





ww^R only NPDA is possible

NPDA ⊃ DPDA

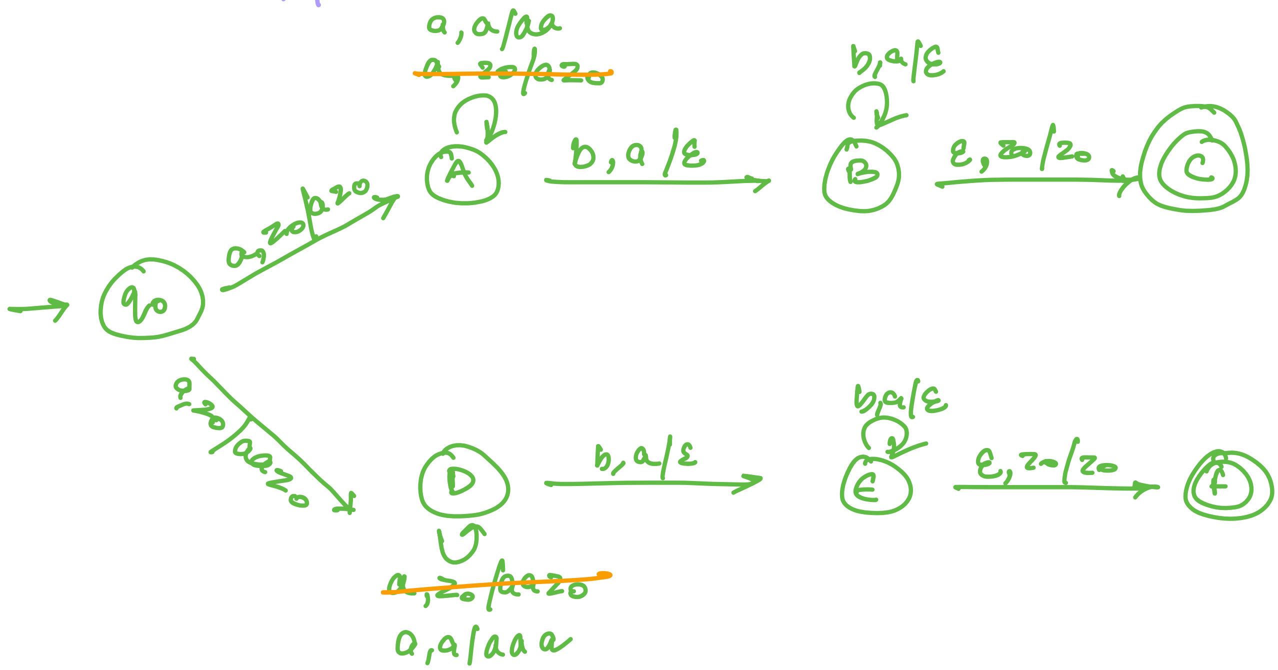
NPDA is more powerful than DPDA

DFA ≅ NFA
Equally
powerful

Eg: $L = \{a^n b^n \mid n \geq 1\} \cup \{a^n b^{2n} \mid n \geq 1\}$

a: push a
b: pop a

a: push 2a's
b: pop 1 a



Eg: $\{a^i b^j c^k d^l \mid i=k \text{ or } j=l\}$ $\quad i,j,k,l \geq 1$

no of a's = no of c's

OR

no. of b's = no of d's

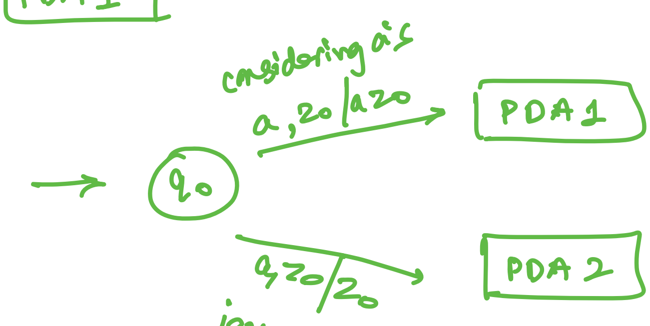
Rewrite: $\{a^m b^j c^m d^l\} \cup \{a^i b^m c^k d^m\}$

Push a's
ignore b's
for c, pop a
ignore d

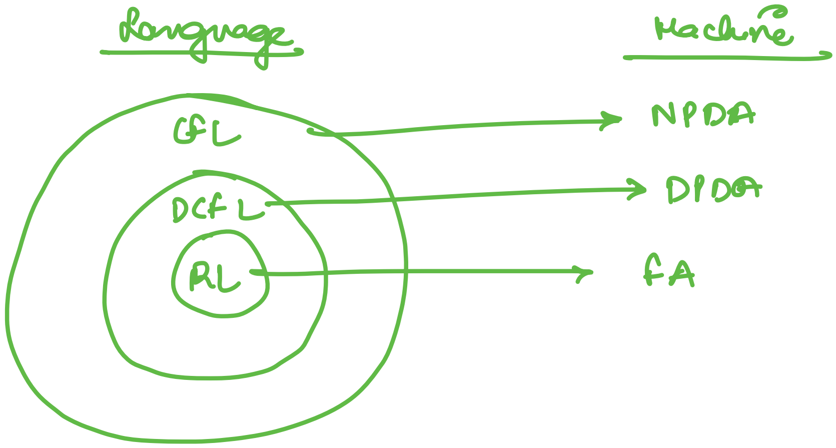
PDA 1

ignore a's
push b's
ignore c's
for d, pop b

PDA 2



growing a's



Eg: $a^{m+n} b^n c^m \mid n,m \geq 1$

$a^m a^n b^n c^m$

Reg ✗
DCFL ✓
CFL ✓

Eg: $a^m b^{m+n} c^n \mid n,m \geq 1$

$a^m b^m b^n c^n$

RL ✗
DCFL ✓
CFL ✓

Eg: $a^m b^n c^{m+n} \mid n,m \geq 1$

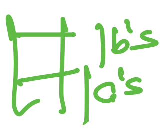
$a^m b^n c^n c^m$

RL ✗
DCFL ✓
CFL ✓

Eg: $a^m b^m c^n d^n \mid m,n \geq 1$

RL ✗
DCFL ✓
CFL ✓

Eg: $\underset{\text{push}}{\underline{a^m}}\ \underset{\text{push}}{\underline{b^n}}\ c^m d^n \mid m,n \geq 1$



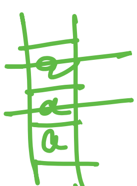
RL ✗
DCFL ✗
CFL ✗

Eg: $a^m b^n c^n d^m \mid m,n \geq 1$

push push pop pop

RL ✗
DCFL ✓
CFL ✓

Eg: $a^m b^n \mid m > n$



$\varepsilon, a \rightarrow$ final state

RL ✗
DCFL ✓
CFL ✓

Eg: $a^n b^{2n} \mid n \geq 1$

RL ✗
DCFL ✓
CFL ✓

Eg: $a^n b^{n^2} \mid n \geq 1$

$n=3$ $a^3 b^9$ aaa bbb bbb bbb → every a: push 3a's

$n=4$ $a^4 b^{16}$ aaaa bbbb bbbb bbbb bbbb → every a: push 4a's

RL ✗ DCFL ✗ CFL ✗

Eg: $a^n b^{2^n} \mid n \geq 1$

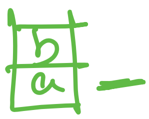
RL ✗
DCFL ✗
CFL ✗

Eg: $wcw^R \mid w \in (a,b)^*$

RL ✗
DCFL ✗
CFL ✓

Eg: $ww \mid w \in (a,b)^*$

ab ab



R ✗
DCFL ✗
CFL ✗

Eg: $a^n b^n c^m \mid n > m$

RL ✗
DCFL ✗
CFL ✗

Eg: $a^n b^n c^n d^n \mid \underline{n \leq 10^{10}}$ → upperbound

Reg ✓
DCFL ✓
CFL ✓

Eg: $a^n b^{2n} c^{3n} \mid n \geq 1$

1a → push 3a's
1b → ~~push~~ pop 1a
1c → pop 1a

$a^2 b^2 c^8$

~~aaaaaaaaaa~~

Reg ✗
DCFL ✗
CFL ✗

Eg: $xcy \mid x, y \in (0,1)^*$

c should be present as a substring

Reg ✓
DCFL ✓
CFL ✓

Eg: $xx^R \mid x \in (a,b)^*, |x| = l$

$\underbrace{\overset{a/b}{\_}\ \overset{a/b}{\_}\ \overset{x}{\overset{a/b}{\_}}\ \_\ \_}_{l}\ \underbrace{\_\ \_\ \_\ \_\ \_}_{l}^{x^R}$

$2^{2l} = 4^l \mid$ finite

R ✓
DCFL ✓
CFL ✓

Eg: $\underline{wcw}\, w^R \mid w \in (a,b)^*$

CFL ✗

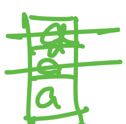
R ✗
DCFL ✗
CFL ✗

Eg: $a^n b^{3^n} \mid n \geq 1$

RL ✗
DCFL ✗
CFL ✗

Eg: $a^m b^n \mid m \neq n$

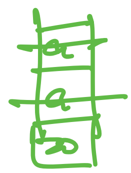
a^3b^2 $m > n$



ϵ, a

$m < n$

a^2b^3



b, z_0

RL ✗
DCFL ✓
CFL ✓

Eg: $a^m b^n \mid m = 2n+1$

$a^{2n+1} b^n$

RL ✗
DCFL ✓
CFL ✓

Eg: $a^i b^{2j} \mid i \neq 2j+1$

$i > 2j+1$

$i < 2j+1$

RL ✗
DCFL ✓
CFL ✓

Eg: $a^{2^n} \mid n \geq 1$

Eg: $a^{n!} \mid n \geq 1$

Eg: $a^m \mid m$ is prime

RL ✗
DCFL ✗
CFL ✗

Eg: $a^k \mid k$ is even

$(a^0, a^2, a^4 \ldots)$

RL ✓
DCFL ✓
CFL ✓

Eg: $a^i b^j c^k \mid i > j > k$

PDA can't handle 2 comparisons

RL ✗
DCFL ✗
CFL ✗

Eg: $a^i b^j c^k \mid j = i + k$

$a^i b^{i+k} c^k = a^i b^i b^k c^k$

RL ✗
DCFL ✓
CFL ✓

Eg: $a^i b^j c^k d^l \mid i = k$ or $j = l$

RL ✗
DCFL ✗
CFL ✓

Eg: $a^i b^j c^k d^l \mid i = k$ and $j = l$

$a^m b^n c^m d^n$

RL ✗
DCFL ✗
CFL ✗

Eg: $a^m b^l c^k d^n \mid m, l, k, n \geq 1$

$aa^* bb^* cc^* dd^*$

RL ✓
DCFL ✓
CFL ✓

Eg: $a^n b^{4m} \mid n, m \geq 1$

$aa^* (bbbb)(bbbb)^*$

RL ✓
DCFL ✓
CFL ✓

Eg: $a^{2n+1} \mid n \geq 1$

odd no of a's

RL ✓
DCFL ✓
CFL ✓

Eg: $a^{n^n} \mid n \geq 1$

RL ✗ CFL ✗
DCFL ✗

Eg:- $w \mid w \in (a,b)^*$ $|w| \geq 100$

$\underbrace{(a+b)(a+b) \text{———————} (a+b)}_{100}(a+b)^*$

RL ✓
DCFL ✓
CFL ✓

Eg: $w \mid w \in (a,b,c)^*$ $n_a(w) = n_b(w) = n_c(w)$

3 cmp ✗

RL ✗
DCFL ✗
CFL ✗

CFL

Generator → CFG

Acceptor → PDA → DPDA, NPDA

$CFG \rightleftarrows PDA$ Both are equivalent in power.

CFG to PDA:

1. Convert CFG productions to GNF
   - $NT \to T$
   - $NT \to T(NT)^*$

2. PDA will have only 1 state $\{q\}$

3. Start Symbol of CFG will be initial symbol in PDA

S →

Stack

4. For non terminal symbols (variables), add the following rule

$$A \rightarrow xYZ$$

$$\delta(\underset{\text{State}}{q}, \underset{\text{Input Alphabet}}{\varepsilon}, \underset{\text{Top of Stack}}{A}) \longrightarrow (\underset{\text{State}}{q}, \underset{\text{new top of stack}}{xYZ})$$

5. For each terminal symbol 'x', add the following rule:

$$\delta(\underset{\text{State}}{q}, \underset{\text{Input alphabet}}{x}, \underset{\text{top of Stack}}{x}) \longrightarrow (\underset{\text{State}}{q}, \underset{\text{pop the element}}{\varepsilon})$$

Q: Construct a PDA equivalent to following CFG productions?

$$S \longrightarrow aAA$$

$$A \rightarrow aS \mid bS \mid a$$

1. CFG $\longrightarrow$ GNF : Already in GNF
2. $\{q\}$
3. S
4. $\delta(q, \varepsilon, S) \longrightarrow (q, aAA)$

   $\delta(q, \varepsilon, A) \longrightarrow (q, aS) \mid (q, bS) \mid (q, a)$

5. $\delta(q, a, a) \longrightarrow (q, \varepsilon)$

   $\delta(q, b, b) \longrightarrow (q, \varepsilon)$

PDA $\longrightarrow$ CFG

PDA : $Q, \Sigma, \delta, q_0, z_0, f, \Gamma$

Set of States, input alphabets, Transition, initial state, bottom of stack, final states, Stack Alphabets

Grammar:

$$NT \longrightarrow S \cup [q, A, p]$$

triplet

$q, p \in Q$

$A \in \Gamma$

1. $S \longrightarrow [q_0, z_0, p]$ for each p

2. $\delta(q, x, A) = (p, B_1, B_2 \; .... \; B_m)$

production

$$[q, A, q_{m+1}] \longrightarrow x \, [p, B_1, q_2][q_2, B_2, q_3] \; .... \; [q_m, B_m, q_{m+1}]$$

3. $\delta(q, x, A) = (p, \varepsilon)$

$$[q, A, p] \longrightarrow x$$

$x \in \Sigma \cup \{\varepsilon\}$

input alphabet, union, epsilon

Example:

$$\left\{ \overset{Q}{\{q_0, q_1\}}, \; \overset{\Sigma}{\{a, b\}}, \; \overset{\delta}{\delta}, \; \overset{q_0}{q_0}, \; \overset{z_0}{z_0}, \; \overset{f}{\phi}, \; \overset{\Gamma}{\{z_0, x\}} \right\}$$

PDA

acceptance by final state $\approx$ acc. by empty stack

$$
\left.
\begin{aligned}
&\checkmark\ \delta(q_0, a, z_0) = (q_0, xz_0)\\
&\checkmark\ \delta(q_0, a, x) = (q_0, xx)\\
&\checkmark\ \delta(q_0, b, x) = (q_1, \varepsilon)\\
&\delta(q_1, b, x) = (q_1, \varepsilon)\\
&\delta(q_1, \varepsilon, z_0) = (q_1, \varepsilon)
\end{aligned}
\right\} a^n b^n \mid n \geq 1
$$

1. $S \rightarrow [q_0, z_0, P]$ for each P, $P \in Q$

$Q : \{q_0, q_1\}$

$P \rightarrow q_0$

$\rightarrow q_1$

$S \rightarrow [q_0, z_0, q_0]$

$S \rightarrow [q_0, z_0, q_1]$

2. $\delta(q, x, A) = (P, B_1 B_2 \cdots B_m)$

production

$[q, A, q_{m+1}] \longrightarrow x\, [P, B_1, q_2][q_2, B_2, q_3] \cdots [q_m, B_m, q_{m+1}]$

$$\delta(\overset{q}{q_0}, \overset{x}{a}, \overset{A}{z_0}) = (\overset{P}{q_0}, \overset{B_1}{x}\overset{B_2}{z_0})$$

$[q_0, z_0, \_\ ] = a\, [q_0, x, \_\ ][\ \_, z_0, \_\ ]$

all possible combinations

$$[q_0, z_0, q_0] = a\,[q_0, x, q_0]\,[q_0, z_0, q_0]$$
$$[q_0, z_0, q_0] = a\,[q_0, x, q_1]\,[q_1, z_0, q_0]$$
$$[q_0, z_0, q_1] = a\,[q_0, x, q_0]\,[q_0, z_0, q_1]$$
$$[q_0, z_0, q_1] = a\,[q_0, x, q_1]\,[q_1, z_0, q_1]$$

$$\delta(\overset{q}{q_0}, \overset{x}{a}, \overset{A}{x}) = (\overset{P}{q_0}, \overset{B_1}{x}\overset{B_2}{x})$$

$$[q_0, x, \_\,] = a\,[q_0, x, \_\,]\,[\_\,, x, \_\,]$$

$$[q_0, x, q_0] = a\,[q_0, x, q_0]\,[q_0, x, q_0]$$
$$[q_0, x, q_0] = a\,[q_0, x, q_1]\,[q_1, x, q_0]$$
$$[q_0, x, q_1] = a\,[q_0, x, q_0]\,[q_0, x, q_1]$$
$$[q_0, x, q_1] = a\,[q_0, x, q_1]\,[q_1, x, q_1]$$

3. $\delta(q, x, A) = (P, \varepsilon)$

$$[q, A, P] \rightarrow x$$

$$\overset{q}{}\ \overset{x}{}\ \overset{A}{} \qquad \overset{P}{}\ \overset{\varepsilon}{}$$

$\delta(q_0, b, X) = (q_1, \varepsilon) \longrightarrow [q_0, X, q_1] \rightarrow b$

$\delta(\overset{q}{q_1}, \overset{x}{b}, \overset{A}{X}) = (\overset{p}{q_1}, \overset{\varepsilon}{\varepsilon}) \longrightarrow [q_1, X, q_1] \rightarrow b$

$\delta(\overset{q}{q_1}, \overset{x}{\varepsilon}, \overset{A}{z_0}) = (\overset{p}{q_1}, \overset{\varepsilon}{\varepsilon}) \longrightarrow [q_1, z_0, q_1] \rightarrow \varepsilon$

$S \rightarrow [q_0, z_0, q_0]$

$S \rightarrow [q_0, z_0, q_1]$

$[q_0, z_0, q_0] = a\,[q_0, X, \underline{q_0}]\,[\underline{q_0}, z_0, q_0]$

$[q_0, z_0, q_0] = a\,[q_0, X, \underline{q_1}]\,[\underline{q_1}, z_0, q_0]$

$[q_0, z_0, q_1] = a\,[q_0, X, q_0]\,[q_0, z_0, q_1]$

$[q_0, z_0, q_1] = a\,[q_0, X, q_1]\,[q_1, z_0, q_1]$

$[q_0, X, q_0] = a\,[q_0, X, q_0]\,[q_0, X, q_0]$

$[q_0, X, q_0] = a\,[q_0, X, q_1]\,[q_1, X, q_0]$

$[q_0, X, q_1] = a\,[q_0, X, q_0]\,[q_0, X, q_1]$

$[q_0, X, q_1] = a\,[q_0, X, q_1]\,[q_1, X, q_1]$

$[q_0, X, q_1] \rightarrow b$

$[q_1, X, q_1] \rightarrow b$

$[q_1, z_0, q_1] \rightarrow \varepsilon$

Remove useless symbols

Triplet which is present on RHS of production but not present in LHS.

$[q_1, z_0, q_0]$

$S \rightarrow [q_0, z_0, q_0]$

$S \rightarrow [q_0, z_0, q_1]$

$[q_0, z_0, q_0] = a\, [q_0, x, q_0]\, [q_0, z_0, q_0]$

$\cancel{[q_0, z_0, q_0] = a\, [q_0, x, q_1]\, [q_1, z_0, q_0]}$

$[q_0, z_0, q_1] = a\, [q_0, x, q_0]\, [q_0, z_0, q_1]$

$[q_0, z_0, q_1] = a\, [q_0, x, q_1]\, [q_1, z_0, q_1]$

$[q_0, x, q_0] = a\, [q_0, x, q_0]\, [q_0, x, q_0]$

$[q_0, x, q_0] = a\, [q_0, x, q_1]\, [q_1, x, q_0]$

$[q_0, x, q_1] = a\, [q_0, x, q_0]\, [q_0, x, q_1]$

$[q_0, x, q_1] = a\, [q_0, x, q_1]\, [q_1, x, q_1]$

$[q_0, x, q_1] \rightarrow b$

$[q_1, x, q_1] \rightarrow b$

$[q_1, z_0, q_1] \rightarrow \varepsilon$

$[q_1, x, q_0]$

$S \rightarrow [q_0, z_0, q_0]$

$S \rightarrow [q_0, z_0, q_1]$

$[q_0, z_0, q_0] = a\, [q_0, x, q_0]\, [q_0, z_0, q_0]$

$\cancel{[q_0, z_0, q_0] = a\, [q_0, x, q_1]\, [q_1, z_0, q_0]}$

$[q_0, z_0, q_1] = a\, [q_0, x, q_0]\, [q_0, z_0, q_1]$

$[q_0, z_0, q_1] = a\, [q_0, x, q_1]\, [q_1, z_0, q_1]$

$[q_0, x, q_0] = a\, [q_0, x, q_0]\, [q_0, x, q_0]$

$\cancel{[q_0, x, q_0] = a\, [q_0, x, q_1]\, [q_1, x, q_0]}$

$[q_0, x, q_1] = a\, [q_0, x, q_0]\, [q_0, x, q_1]$

$[q_0, x, q_1] = a\, [q_0, x, q_1]\, [q_1, x, q_1]$

$[q_0, x, q_1] \to b$

$[q_1, x, q_1] \to b$

$[q_1, z_0, q_1] \to \varepsilon$

(q_0, x, q_0)

$A \to a\underline{A}\,\underline{A}$

$\to aa\underline{AA}\,A$

$S \to [q_0, z_0, q_0]$

$S \to [q_0, z_0, q_1]$

$\cancel{[q_0, z_0, q_0] = a[q_0, x, q_0][q_0, z_0, q_0]}$

$\cancel{[q_0, z_0, q_0] = a[q_0, x, q_1][q_1, z_0, q_0]}$

$\cancel{[q_0, z_0, q_1] = a[q_0, x, q_0][q_0, z_0, q_1]}$

$[q_0, z_0, q_1] = a[q_0, x, q_1][q_1, z_0, q_1]$

$\cancel{\overset{A}{[q_0, x, q_0]} = a\overset{A}{[q_0, x, q_0]}\overset{A}{[q_0, x, q_0]}}$

$\cancel{[q_0, x, q_0] = a[q_0, x, q_1][q_1, x, q_0]}$

$\cancel{[q_0, x, q_1] = a[q_0, x, q_0][q_0, x, q_1]}$

$[q_0, x, q_1] = a[q_0, x, q_1][q_1, x, q_1]$

$[q_0, x, q_1] \to b$

$[q_1, x, q_1] \to b$

$[q_1, z_0, q_1] \to \varepsilon$

(q_0, z_0, q_0)

$\cancel{S \to [q_0, z_0, q_0]}$

$S \to [q_0, z_0, q_1]$

$\cancel{[q_0, z_0, q_0] = a[q_0, x, q_0][q_0, z_0, q_0]}$

$\cancel{[q_0, z_0, q_0] = a[q_0, x, q_1][q_1, z_0, q_0]}$

$\cancel{[q_0, z_0, q_1] = a[q_0, x, q_0][q_0, z_0, q_1]}$

$[q_0, z_0, q_1] = a[q_0, x, q_1][q_1, z_0, q_1]$

$\cancel{[q_0, x, q_0] = a[q_0, x, q_0][q_0, x, q_0]}$

$\cancel{[q_0, x, q_0] = a[q_0, x, q_1][q_1, x, q_0]}$

$\cancel{[q_0, x, q_1] = a[q_0, x, q_0][q_0, x, q_1]}$

$[q_0, x, q_1] = a\,[q_0, x, q_1]\,[q_1, x, q_1]$

$[q_0, x, q_1] \to b$

$[q_1, x, q_1] \to b$

$[q_1, z_0, q_1] \to \varepsilon$

Final Productions:

$S \to \underbrace{[q_0, z_0, q_1]}_{A}$

$A: \quad \underbrace{[q_0, z_0, q_1]}_{A} = a\,\underbrace{[q_0, x, q_1]}_{B}\,\underbrace{[q_1, z_0, q_1]}_{C}$

$B: \quad \underbrace{[q_0, x, q_1]}_{B} = a\,\underbrace{[q_0, x, q_1]}_{B}\,\underbrace{[q_1, x, q_1]}_{D}$

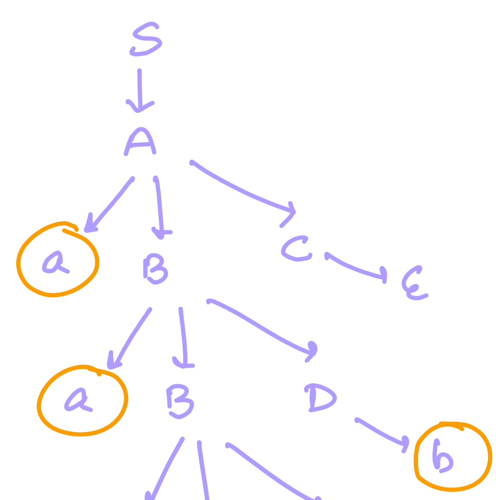
$B: \quad \underbrace{[q_0, x, q_1]}_{B} \to b$

$D: \quad \underbrace{[q_1, x, q_1]}_{D} \to b$

$\underbrace{[q_1, z_0, q_1]}_{C} \to \varepsilon$

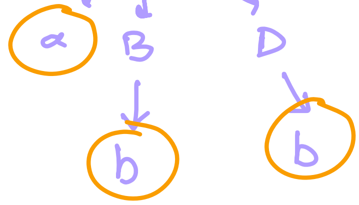
$\downarrow$

$$\left.\begin{aligned} S &\to A \\ A &\to aBC \\ B &\to aBD \\ B &\to b \\ D &\to b \\ C &\to \varepsilon \end{aligned}\right\} \text{Context free Grammar}$$



$a^n b^n \mid n \geq 1$

$a^3 b^3$



$$\text{CFG} \rightleftarrows \text{PDA}$$

CFG & PDA are equivalent in power.